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Fermion coupling to Chern–Simons theories with higher-order Lagrangian in (2 + 1) dimensions

A Greco†, C Repetto‡, O P Zandron‡ and O S Zandron‡
Facultad de Ciencias Exactas Ingeniería y Agrimensura de la UNR, Av. Pellegrini 250-2000 Rosario, Argentina

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Abstract. The classical and quantum formalism for the constrained Hamiltonian system with singular higher-order Lagrangian describing the Fermion coupling to Chern–Simons theories in (2 + 1) dimensions is constructed. We perform the canonical and path-integral quantizations for the Abelian case. The non-Abelian case is also discussed.

1. Introduction

Different quantum field theories in (2+1) dimensions have been investigated with increasing interest in the past few years. Several interesting problems are present in the (2 + 1) dimensional planar physics [1]. For instance, for some time it is known how charged planar matter interacting with 'photons' whose dynamics is governed by the Maxwell Lagrangian plus a Chern–Simons (CHS) term, gives rise to topologically massive (2 + 1)-dimensional electrodynamics [2]. The addition of the CHS term to the Maxwell action leads to a modified Gauss law with the important consequence that any charged excitation also carries a magnetic flux, which is proportional to the charge.

More recently the quantum mechanics coupled to the gauge field, which has the CHS term as the action, was considered [3]. The generalized Hamiltonian formalism was constructed and the canonical and the path-integral quantizations were performed. The CP^1 model with the CHS term coupled to a charged fermion, as a possible model for the high T_c superconductivity was also treated [4]. In [4], the canonical and the path-integral quantization methods for this coupled system were developed and the Bose–Fermi statistics transmutation was discussed.

Moreover, pure U(1) and SU(N) CHS theories and topologically massive theory in (2+1) dimensions were quantized by means of the Dirac formalism [5]. The constraint structure and the symmetry properties of the dynamical system were analysed.

The dynamical unitary and possible renormalizable topologically massive three-dimensional gravity was also investigated [6].

On the other hand, the conformal supergravity in (2+1) dimensions can be described by a CHS term [7]. The conformal gravity models present two important features. First, these models show that local symmetry can exist in flat space-time and moreover, the conformal gravity in three dimensions is finite and exactly soluble [8]. Second, the requirement of complete invariance under all local symmetries implies constraints on curvatures, and

† Member of Consejo de Investigaciones de la UNR.
‡ Members of Consejo Nacional de Investigaciones Científicas y Técnicas-Argentina.

consequently the higher-derivative character of the conformal theory is made evident when the second-order formalism is carried out. Hence, due to the appearance of higher derivatives, the definition of canonical variables and the construction of the Dirac formalism for this constrained Hamiltonian system is non-trivial [9].

The dynamical systems described in terms of higher derivatives have been studied by several authors [10] and constitute an interesting problem of current research in quantum field theory.

As already mentioned above, the interest to consider higher derivative terms exists because of the possible application of these models to high T_c superconductivity.

Other motivation for considering this kind of theory is due to the fact that by adding terms highly derived in the Lagrangian, they can be used for regularizing ultraviolet divergences in the quantum theory. Likewise, this compels us to reconsider the renormalizability problem in these models. A complete answer about this question only could be given once the diagrammatic of the model is done. We can expect, as it occurs in the usual interacting CHS theories with matter [11], that in our case the theory also can be renormalized in some regimes.

Another important question, not treated in the present paper, is the unitarity problem. It is well known that in higher derivative gauge theories it is possible that the unitarity can be violated when ghost states with negative norm are present. This problem is related with the form of the effective interacting Lagrangian and the expression of the effective propagator of the boson field highly derived.

To solve these two questions we need to make known the Feynman rules and to construct the diagrammatic. In a forthcoming advanced paper, we treat extensively these topics with the aim of giving, as well as possible, an answer to these questions.

Therefore, in the present paper we begin by constructing the classical and quantum Dirac formalism for the constrained Hamiltonian system with singular higher-order Lagrangian describing the fermion coupling to CHS theories in $(2+1)$ dimensions. We perform the canonical and the path-integral quantizations. This last approach is accomplished by extending the Faddeev–Senjanovic method [12] to the higher-derivative case.

The paper is organized as follows. In section 2, we construct the classical generalized Hamiltonian formalism for the Abelian case, working as closely as possible to the Dirac prescriptions. By means of the Ostrogradski transformation [13] the momenta are introduced and so, the primary constraints remain defined and the extended Hamiltonian can be written. In section 3, after the complete set of constraints are analysed and classified, the Dirac brackets can be found. Next we perform the canonical quantization. In section 4, we carry out the path-integral quantization by extending the Faddeev–Senjanovic method to the higher-derivative system under consideration. In section 5, we discuss the generalization of the method to the non-Abelian case.

2. Classical generalized Hamiltonian formalism

Our starting point is to consider the matter coupling to Abelian CHS theories in $(2+1)$ dimensions. The system is described by a singular Lagrangian density containing higher-derivative terms given by:

$$\mathcal{L} = \mathcal{L}_{\text{top}} + \mathcal{L}_h + \mathcal{L}_f + \mathcal{L}_{\text{int}} \tag{2.1}$$

where $\mathcal{L}_{\text{top}}$ is the electromagnetic Lagrangian density with a topological mass term, i.e. a CHS term, and it is given by

$$\mathcal{L}_{\text{top}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4\pi}\varepsilon^{\mu\nu\rho}\partial_\mu A_\nu A_\rho. \tag{2.2a}$$

The other pieces of the total Lagrangian density are written:

$$\mathcal{L}_h = -\frac{c}{4\pi}\partial_\rho F_{\mu\nu}\partial^\rho F^{\mu\nu} \tag{2.2b}$$

$$\mathcal{L}_f = i\left(\frac{a+1}{2}\right)\bar{\psi}\gamma^\mu\partial_\mu\psi + i\left(\frac{a-1}{2}\right)\partial_\mu\bar{\psi}\gamma^\mu\psi - m\bar{\psi}\psi \tag{2.2c}$$

$$\mathcal{L}_{\rm int} = e\bar{\psi}\gamma^\mu\psi A_\mu. \tag{2.2d}$$

The field strength tensor is written in terms of potentials in the usual way $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ($\mu, \nu = 0, 1, 2$ denoting the space-time components), c is a dimensional coupling constant and κ is the topological mass of the gauge field. The kinetic fermionic term is included in the general form by using the parameter a [14]. Throughout this paper we use the convention $\varepsilon^{012} = \varepsilon^{12} = 1$, the Minkowski metric $g_{\mu\nu} = \mathrm{diag}(1, -1, -1)$ and the Dirac γ-matrices $\gamma^0 = \sigma^3$, $\gamma^1 = i\sigma^1$ and $\gamma^2 = i\sigma^2$ (σs are the Pauli matrices).

Let us consider the following independent dynamical field variables A_μ, $B_\mu = \dot{A}_\mu$ $\psi_{(\alpha)}$ and $\bar{\psi}_{(\alpha)}$. By means of the Ostrogradski transformation the following canonical momenta can be introduced:

$$P^\mu = \frac{\partial\mathcal{L}}{\partial\dot{A}_\mu} - \partial_\alpha\frac{\partial\mathcal{L}}{\partial(\partial_\alpha B_\mu)} \tag{2.3a}$$

$$Q^\mu = \frac{\partial\mathcal{L}}{\partial\dot{B}_\mu} \tag{2.3b}$$

$$\bar{\Pi}^{(\alpha)} = \frac{\partial\mathcal{L}}{\partial\dot{\psi}_{(\alpha)}} \tag{2.3c}$$

$$\Pi^{(\alpha)} = \frac{\partial\mathcal{L}}{\partial\dot{\bar{\psi}}_{(\alpha)}} \tag{2.3d}$$

where the Poisson brackets for canonical conjugate variables are given by

$$[A_\mu(x,t), P^\nu(y,t)]_{PB} = -[P^\nu(y,t), A_\mu(x,t)]_{PB} = \delta^\nu_\mu\delta(x-y) \tag{2.4a}$$

$$[B_\mu(x,t), Q^\nu(y,t)]_{PB} = -[Q^\nu(y,t), B_\mu(x,t)]_{PB} = \delta^\nu_\mu\delta(x-y) \tag{2.4b}$$

$$[\psi_{(\alpha)}(x,t), \bar{\Pi}^{(\beta)}(y,t)]_{PB} = [\bar{\Pi}^{(\beta)}(y,t), \psi_{(\alpha)}(x,t)]_{PB} = -\delta^{(\beta)}_{(\alpha)}\delta(x-y) \tag{2.4c}$$

$$[\bar{\psi}_{(\alpha)}(x,t), \Pi^{(\beta)}(y,t)]_{PB} = [\Pi^{(\beta)}(y,t), \bar{\psi}_{(\alpha)}(x,t)]_{PB} = -\delta^{(\beta)}_{(\alpha)}\delta(x-y) \tag{2.4d}$$

while all the others vanish. From now on the time t is omitted from the arguments.

By writing in equations (2.3a, b) separately the time component and the spatial ones:

$$P^0 = -\frac{c}{\pi}\partial_0\partial_i F^{0i} \tag{2.5a}$$

$$P^i = F^{i0} + \frac{\kappa}{4\pi}\varepsilon^{ij}A_j + \frac{c}{\pi}(\nabla^2 F^{0i} + \partial_0\partial_j F^{ji}) - \partial_0 Q^i \tag{2.5b}$$

$$Q^0 = 0 \tag{2.5c}$$

$$Q^i = -\frac{c}{\pi}\partial_0 F^{0i} \tag{2.5d}$$

we can see that these equations give rise to the following primary constraints

$$\Phi^{(0)}(x) = Q^0(x) \approx 0 \tag{2.6a}$$

$$\Phi_{(\alpha)}(x) = \Pi_{(\alpha)}(x) - i\left(\frac{a-1}{2}\right)\gamma_0\psi_{(\alpha)} \approx 0 \tag{2.6b}$$

$$\bar{\Phi}_{(\alpha)}(x) = \bar{\Pi}_{(\alpha)}(x) + i\left(\frac{a+1}{2}\right)\bar{\psi}_{(\alpha)}\gamma_0 \approx 0. \tag{2.6c}$$

By means of the momenta given in equation (2.3), the canonical Hamiltonian density remains defined as follows:

$$\mathcal{H}_{\text{can}} = B_\mu P^\mu + \dot{B}_\mu Q^\mu + \dot{\bar{\psi}}_{(\alpha)}\Pi^{(\alpha)} + \bar{\Pi}^{(\alpha)}\dot{\psi}_{(\alpha)} - \mathcal{L}. \tag{2.7}$$

Once the Lagrangian density (2.1) is used and the velocities $\dot{B}_\mu$, $\dot{\psi}_{(\alpha)}$ and $\dot{\bar{\psi}}_{(\alpha)}$ are eliminated, the final expression for $\mathcal{H}_{\text{can}}$ is:

$$\begin{aligned}\mathcal{H}_{\text{can}} = {} & B_\mu P^\mu - \frac{\pi}{2c}Q_i Q^i + \partial_i B_0 Q^i + \frac{1}{4}F_{ij}F^{ij} + \frac{1}{2}B_i B^i + \frac{1}{2}\partial_i A_0 \partial^i A_0 - B_i \partial^i A_0 \\ & + \frac{c}{4\pi}\partial_i F_{jk}\partial^i F^{jk} + \frac{c}{4\pi}\partial_0 F_{ij}\partial^0 F^{ij} + \frac{c}{2\pi}\partial_k F_{0j}\partial^k F^{0j} - \frac{\kappa}{4\pi}B_i A_j \varepsilon^{ij} \\ & + \frac{\kappa}{4\pi}\partial_i A_0 A_j \varepsilon^{ij} - \frac{\kappa}{4\pi}\partial_i A_j A_0 \varepsilon^{ij} - i\left(\frac{a+1}{2}\right)\bar{\psi}\gamma^i\partial_i\psi \\ & - i\left(\frac{a-1}{2}\right)\partial_i\bar{\psi}\gamma^i\psi + m\bar{\psi}\psi - e\bar{\psi}\gamma^\mu\psi A_\mu. \end{aligned} \tag{2.8}$$

Now, from equations (2.6) and (2.7) we can write the extended Hamiltonian (first-class dynamical quantity):

$$H_T = \int \mathrm{d}^2x\, \mathcal{H}_T \tag{2.9}$$

generator of time evolutions of generic functionals. The Hamiltonian density $\mathcal{H}_T$ remains defined by:

$$\mathcal{H}_T = \mathcal{H}_{\text{can}} + \delta\Phi^{(0)} + \bar{\lambda}_{(\alpha)}\Phi^{(\alpha)} + \bar{\Phi}^{(\alpha)}\lambda_{(\alpha)} \tag{2.10}$$

where δ is a bosonic Lagrange multiplier, and $\lambda_{(\alpha)}$ and $\bar{\lambda}_{(\alpha)}$ are fermionic.

Now, we must go on with the Dirac's algorithm and impose the consistency conditions on the constraints $\Phi^{(k-1)}$ according to $\Phi^{(k)} = \dot{\Phi}^{(k-1)} = [\Phi^{(k-1)}, H_T] \approx 0$. Hence for the bosonic constraint $\Phi^{(0)}$ we find the following secondary constraints:

$$\Phi^{(1)} = \dot{\Phi}^{(0)} = [\Phi^{(0)}, H_T]_{PB} = -P^0 + \partial_i Q^i \approx 0 \tag{2.11a}$$

$$\Phi^{(2)} = \dot{\Phi}^{(1)} = [\Phi^{(1)}, H_T]_{PB} = -\partial_i P^i - \frac{\kappa}{4\pi}\partial_j A_k \varepsilon^{jk} - e\bar{\psi}\gamma^0\psi \approx 0 \tag{2.11b}$$

and the consistency for $\Phi^{(2)}(x)$ is automatically satisfied.

Next the consistency conditions for the two fermionic constraints $\Phi_{(\alpha)}(x)$ and $\bar{\Phi}_{(\alpha)}(x)$ determine the Lagrange multipliers $\bar{\lambda}_{(\alpha)}$ and $\lambda_{(\alpha)}$ respectively and they are written:

$$\bar{\lambda}_{(\alpha)} = -(\partial_i\bar{\psi}\gamma^i\gamma^0)_{(\alpha)} + im(\bar{\psi}\gamma^0)_{(\alpha)} - ieA_\mu(\bar{\psi}\gamma^\mu\gamma^0)_{(\alpha)} \tag{2.12a}$$

$$\lambda_{(\alpha)} = (\gamma^0\gamma^i\partial_i\psi)_{(\alpha)} + im(\gamma^0\psi)_{(\alpha)} - ieA_\mu(\gamma^0\gamma^\mu\psi)_{(\alpha)}. \tag{2.12b}$$

At this point by computing the Poisson brackets among the constraints it is easy to conclude that the constraints $\Phi^{(0)}(x)$ and $\Phi^{(1)}(x)$ are first-class while the constraints $\Phi^{(2)}(x)$, $\Phi_{(\alpha)}$ and $\bar{\Phi}_{(\alpha)}$ are second-class. So, the constraints $\Phi^{(0)}(x)$ and $\Phi^{(1)}(x)$ correspond to gauge invariances of the theory under local gauge transformations.

3. Dirac brackets and canonical quantization

As we have seen, the number of second-class constraints is three. Hence, the determinant of the matrix composed of them vanish. That means that they are not independent of each other. As is usual, there is at least one suitable linear combination of the second-class constraints which give rise to another first-class constraint. The new first-class constraint we can find is given by:

$$\Omega(x) = -ie(\bar{\psi}\Pi + \bar{\Pi}\psi) - \frac{\kappa}{4\pi}\partial_j A_k \varepsilon^{jk} - \partial_i P^i \approx 0. \tag{3.1}$$

Therefore, the final set of constraints is given by: (i) the three first-class constraints $\Phi^{(0)}(x)$, $\Phi^{(1)}(x)$ and $\Omega(x)$; (ii) the two second-class constraints $\Phi_{(\alpha)}(x)$ and $\bar{\Phi}_{(\alpha)}(x)$.

Thus, we have arrived at the stage to construct the Dirac brackets which allow us to treat the second-class constraints as strongly equal to zero equations.

The Dirac brackets for variables $O_1(x)$ and $O_2(y)$ are defined by:

$$[O_1(x), O_2(y)]^* = [O_1(x), O_2(y)]_{PB} - [O_1(x), \Psi_a]_{PB}\Delta^{ab}[\Psi_b, O_2(y)]_{PB} \tag{3.2}$$

where the matrix Δ^{ab} is the inverse of the matrix constructed with the elements $[\Psi_a, \Psi_b]_{PB}$ involving the remaining second-class constraints Ψ_a, i.e: $\Delta^{ab}[\Psi_b, \Psi_c]_{PB} = \delta^a_c$, and it results:

$$\Delta = i\begin{pmatrix} 0 & \gamma_0 \\ \gamma_0^T & 0 \end{pmatrix}\delta(x-y). \tag{3.3}$$

Now, by using the definition (3.2) we can obtain the Dirac brackets among dynamical variables. *We write only the non-vanishing Dirac brackets which are modified with respect* to the Poisson ones. That is to say, in the present case, brackets involving only fermionic dynamical variables. So the field-field brackets are:

$$[\bar{\psi}_{(\alpha)}(x), \psi_{(\beta)}(y)]^* = -i(\gamma_0)_{(\beta)(\alpha)}\delta(x-y) \tag{3.4a}$$

$$[\psi_{(\alpha)}(x), \bar{\psi}_{(\beta)}(y)]^* = -i(\gamma_0)_{(\alpha)(\beta)}\delta(x-y). \tag{3.4b}$$

The field-momentum brackets are:

$$[\bar{\psi}_{(\alpha)}(x), \Pi_{(\beta)}(y)]^* = \left(\frac{a-1}{2}\right)\delta_{(\alpha)(\beta)}\delta(x-y) \tag{3.5a}$$

$$[\psi_{(\alpha)}(x), \bar{\Pi}_{(\beta)}(y)]^* = -\left(\frac{a+1}{2}\right)\delta_{(\alpha)(\beta)}\delta(x-y) \tag{3.5b}$$

and finally the momentum-momentum brackets are written:

$$[\bar{\Pi}_{(\alpha)}(x), \Pi_{(\beta)}(y)]^* = \frac{i}{4}(a^2-1)(\gamma_0)_{(\beta)(\alpha)}\delta(x-y) \tag{3.6a}$$

$$[\Pi_{(\alpha)}(x), \bar{\Pi}_{(\beta)}(y)]^* = \frac{i}{4}(a^2-1)(\gamma_0)_{(\alpha)(\beta)}\delta(x-y). \tag{3.6b}$$

Looking at the equation (3.2) we see that the Dirac brackets and the Poisson brackets for the bosonic variables are identical. As was commented above, the system can be canonically quantized by using the Dirac brackets and taking the second-class constraints as strongly equal to zero equations. Then the Hamiltonian system for this higher-derivative theory is described by the following total Hamiltonian:

$$H_T^* = \int \mathrm{d}^2x\,(\mathcal{H}_{\mathrm{can}} + a\Phi^{(0)} + b\Phi^{(1)} + c\Omega) \tag{3.7}$$

where a, b, c are three arbitrary parameters.

The three first-class constraints are written:

$$\Phi^{(0)}(x) = Q^0(x) \approx 0 \tag{3.8a}$$

$$\Phi^{(1)}(x) = -P^0(x) + \partial_i Q^i(x) \approx 0 \tag{3.8b}$$

$$\Omega(x) = -ie(\bar{\psi}_{(\alpha)}(x)\Pi^{(\alpha)}(x) + \bar{\Pi}^{(\alpha)}(x)\psi_{(\alpha)}(x)) - \frac{\kappa}{4\pi}\partial_i A_j(x)\varepsilon^{ij} - \partial_i P^i(x) \approx 0 \tag{3.8c}$$

and they correspond to the gauge symmetries of the system. Now, to complete the quantization, the brackets defined in (2.3a, b), (3.4), (3.5) and (3.6) are replaced into the equal-time (anti) commutators according to the rule:

$$[O_1(x), O_2(y)]_D \to \frac{1}{i\hbar}[\hat{O}_1\hat{O}_2 - (-1)^{|O_1||O_2|}\hat{O}_2\hat{O}_1] \tag{3.9}$$

where $|O_i| = 0$ (or 1) when O_i is bosonic (or fermionic).

Hence, the equal-time (anti) commutators become:

$$[A_\mu(x), P^\nu(y)]_-^* = -\frac{i}{\hbar}\delta_\mu^\nu\delta(x-y) \tag{3.10a}$$

$$[B_\mu(x), Q^\nu(y)]_-^* = -\frac{i}{\hbar}\delta_\mu^\nu\delta(x-y) \tag{3.10b}$$

$$[\psi_{(\alpha)}(x), \bar{\psi}_{(\beta)}(y)]_+^* = \frac{i}{\hbar}(\gamma_0)_{(\alpha)(\beta)}\delta(x-y). \tag{3.10c}$$

We can conclude that the first-class constraints given in equations (3.8) and the corresponding three gauge fixing conditions that we must determine, restrict the phase space variables to the physical one, and so the true Hilbert space is obtained.

4. Path-integral quantization

The path-integral quantization is accomplished according to Faddeev–Senjanovic method [12]. By extending the expression of [12] for the partition function to higher-order theories we obtain:

$$\begin{aligned} Z = \int & \mathrm{d}A_\mu\,\mathrm{d}P^\mu\,\mathrm{d}B_\nu\,\mathrm{d}Q^\nu\,\mathrm{d}\bar{\psi}_{(\alpha)}\,\mathrm{d}\Pi^{(\alpha)}\,\mathrm{d}\psi_{(\beta)}\,\mathrm{d}\bar{\Pi}^{(\beta)}\,\delta(\Phi^{(0)})\delta(\Phi^{(1)})\delta(\Omega)\delta(f_1)\delta(f_2)\delta(f_3) \\ & \times \det[\Phi^{(0)}, \Phi^{(1)}, \Omega, f_1, f_2, f_3]^*\delta(\Phi_{(\alpha)})\delta(\Phi_{(\beta)})\det[\Phi_{(\alpha)}, \Phi_{(\beta)}] \\ & \times \exp i\left[\int \mathrm{d}^3x\,(B_\mu P^\mu + \dot{B}_\nu Q^\nu + \dot{\bar{\psi}}\Pi + \bar{\Pi}\dot{\psi}) - H_T\right]. \end{aligned} \tag{4.1}$$

In all the above expressions $[\alpha_1, \alpha_2, \ldots, \alpha_n]^*$ is the matrix where each component is the Dirac bracket among $\alpha_1, \alpha_2, \ldots, \alpha_n$. With the exception of f_1, f_2 and f_3 all the quantities were defined below. The quantities f_1, f_2 and f_3 are gauge fixing conditions. The gauge fixing conditions must satisfy $\det[f_i \Phi_j]^* \neq 0$ for all first-class constraints Φ_j. Moreover, the gauge fixing conditions f_i must be compatible with the equation of motion.

Let us assume the gauge fixing conditions

$$f_1 = \partial_i A^i \approx 0 \tag{4.2a}$$

$$f_2 = B_0 \approx 0 \tag{4.2b}$$

$$f_3 = \frac{\kappa}{2\pi}\nabla^2 A_0 + e\varepsilon_{ik}\partial^k(\bar{\psi}\gamma^i\psi) + \Box\left(1 - \frac{c}{\pi}\Box\right)\partial_k A_i \varepsilon^{ik} \approx 0 \tag{4.2c}$$

which are admissibles. The condition f_1 is the Coulomb gauge while f_2 and f_3 are consistents with the equation of motion derived from the Lagrangian density (2.1) as can be proved.

The matrix $[\Phi^{(0)}, \Phi^{(1)}, \Omega, f_1, f_2, f_3]^*$ is written as follows:

$$[\Phi^{(0)}, \Phi^{(1)}, \Omega, f_1, f_2, f_3]^* = \begin{pmatrix} 0 & 0 & 0 & 0 & A & 0 \\ 0 & 0 & 0 & 0 & 0 & B \\ 0 & 0 & 0 & C & 0 & D \\ 0 & 0 & -C & 0 & 0 & 0 \\ -A & 0 & 0 & 0 & 0 & 0 \\ 0 & -B & -D & 0 & 0 & 0 \end{pmatrix} \tag{4.3}$$

where A, B, C and D are given by

$$A = -\delta(x - y) \tag{4.4a}$$

$$B = \frac{\kappa}{2\pi}\nabla^2\delta(x - y) \tag{4.4b}$$

$$C = \nabla^2\delta(x - y) \tag{4.4c}$$

$$D = [\Omega(x), f_3(y)]^*. \tag{4.4d}$$

The determinant of the matrix (4.3) does not depend on D and so, it is independent of the field variables and we obtain:

$$\det[\phi^{(0)}, \phi^{(1)}, \Omega, f_1, f_2, f_3]^* = \left(\frac{\kappa}{2\pi}\right)^2 (\nabla^2)^4\delta(x - y) \tag{4.5}$$

and

$$\det[\bar{\Phi}_{(\alpha)}, \Phi_{(\alpha)}] = i\delta(x - y). \tag{4.6}$$

By performing the path-integrals over the fields B_0, P^μ, Q^μ, $\Pi^{(\alpha)}$, $\bar{\Pi}^{(\alpha)}$ we arrive at:

$$Z = \int \mathrm{d}A_\mu\, \mathrm{d}B_i\, \mathrm{d}\bar{\psi}_{(\alpha)}\, \mathrm{d}\psi_{(\beta)}\delta(f_1)\delta(f_3)\exp i[S_{\text{eff}}] \tag{4.7}$$

where the effective action S_{eff} is defined by

$$S_{\text{eff}} = \int \mathrm{d}^3x \left[-\frac{1}{4}F_{ij}F^{ij} - \frac{1}{2}(B_i - \partial_i A_0)(B_i - \partial_i A_0) - \frac{c}{4\pi}(\partial_i F_{jk}\partial^i F^{jk} + \partial_0 F_{ij}\partial^0 F^{ij}) \right.$$
$$- \frac{c}{2\pi}\dot{B}_i\dot{B}^i - \frac{c}{2\pi}\partial_k F_{0j}\partial^k F^{0j} + \frac{\kappa}{4\pi}B_i A_j \varepsilon^{ij} - \frac{\kappa}{4\pi}\partial_i A_0 A_j \varepsilon^{ij} + \frac{\kappa}{4\pi}(\partial_i A_j)A_0\varepsilon^{ij}$$
$$\left. + i\left(\frac{a+1}{2}\right)\bar{\psi}\gamma^\mu\partial_\mu\psi + i\left(\frac{a-1}{2}\right)(\partial_\mu\bar{\psi})\gamma^\mu\psi - m\bar{\psi}\psi + e\bar{\psi}\gamma_\mu\psi A^\mu \right]. \tag{4.8}$$

As it can be seen from the last expression, we have written the quantum problem in terms of a path-integral in which we have four independent fields. This is an important advance because the problem can be treated with all the powerful techniques present in the Feynman path-integral theory. The diagrammatic technique is an interesting example. In principle, it is straightforward to go to the Feynman rules, propagators and vertices, from a path-integral defined in terms of independent fields [15]. It is possible to treat the equation (4.7) for the quantum partition function similarly as it was done for the shell model in the framework of the solid state physics [16]. This equation can be written as follows:

$$Z = \int \mathrm{d}A_\mu\, \mathrm{d}B_i\, \mathrm{d}\bar{\psi}_{(\alpha)}\, \mathrm{d}\psi_{(\beta)}\, \mathrm{d}\Lambda_1\, \mathrm{d}\Lambda_3 \exp i[S^*] \tag{4.9}$$

where

$$S^* = S_{\text{eff}} - \Lambda_1 f_1 - \Lambda_3 f_3 \tag{4.10}$$

and Λ_1, Λ_3 are Lagrange multipliers.

At this stage, from the quantum partition function (4.9), we must recognize propagators and vertices. To do this we can follow the steps of [15]. A bosonic vector quantity X_Σ (where the compound index is $\Sigma = 0, 1, \ldots, 6$) whose components are given by the remaining independent fields A_μ, B_i, Λ_1 and Λ_3 can be defined. So, when the action S^* is written in terms of this vector quantity, it is in principle easy to recognize the propagators defined by the quadratic part of the Lagrangian and the rest is represented by vertices. Consequently, the equation (4.10) can be seen as an effective action for a system that describes the boson vector field X_Σ interacting with a Dirac spinor. As it was commented in the introduction, we are still working in another paper where the Feynman rules and the diagrammatic for this system are treated extensively.

5. Generalization to the non-Abelian case

As we will see the non-Abelian case is different enough to the Abelian one, which relates to the constraint structure. Now, the fields write $\psi = \psi^a t^a$, $A_\mu = A^a_\mu t^a$, $B_\mu = B^a_\mu t^a$ and $F_{\mu\nu} = F^a_{\mu\nu} t^a = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$. The t^a are the generators of the Lie algebra associated to the gauge group SU(N), i.e: $[t^a, t^b] = f^{abc}t^c$, $\mathrm{tr}(t^a t^b) = \delta^{ab}$, $\mathrm{tr}(t^a t^b t^c) = f^{abc}$ and a, b, c denote group representation indices. The field strength components are written

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu. \tag{5.1}$$

To write the Lagrangian density, the trace in the Yang–Mills space must be performed.

Thus, in the non-Abelian case the piece $\mathcal{L}_{\text{top}}$ of the Lagrangian density for topologically massive SU(N) gauge theory (equivalent to the equation (2.2*a*)) is given by:

$$\mathcal{L}_{\text{top}} = -\frac{1}{4}\operatorname{tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{\kappa}{4\pi}\varepsilon^{\mu\nu\rho}\operatorname{tr}\left(\partial_\mu A_\nu A_\rho + \frac{2}{3}A_\mu A_\nu A_\rho\right)$$
$$= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\kappa}{4\pi}\varepsilon^{\mu\nu\rho}\left(\partial_\mu A^a_\nu A^a_\rho + \frac{1}{3}f^{abc}A^a_\mu A^b_\nu A^c_\rho\right). \tag{5.2}$$

Analogously the other parts equivalent to the equations (2.2*b, c, d*) can be written.

The momenta $P^{a\mu}$, $Q^{a\mu}$, $\Pi^a_{(\alpha)}$ and $\bar{\Pi}^a_{(\alpha)}$ are now written as follows:

$$P^{a\mu} = F^{a\mu 0} + \frac{\kappa}{4\pi}\varepsilon^{0\mu\nu}A^a_\nu + \frac{c}{\pi}f^{abc}A_\nu\partial_0 F^{b\mu\nu} - \frac{c}{\pi}\partial_0\partial^0 F^{a\mu 0} - \frac{c}{\pi}\nabla^2 F^{a\mu 0} - \frac{c}{\pi}\partial_i\partial_0 F^{a\mu i} \tag{5.3a}$$

$$Q^{a\mu} = \frac{c}{\pi}\partial_0 F^{a\mu 0} \tag{5.3b}$$

$$\Pi^a_{(\alpha)} = i\left(\frac{a-1}{2}\right)\gamma_0\psi^a_{(\alpha)} \tag{5.3c}$$

$$\bar{\Pi}^a_{(\alpha)} = -i\left(\frac{a+1}{2}\right)\bar{\psi}^a_{(\alpha)}\gamma_0. \tag{5.3d}$$

The three primary constraints are:

$$\Phi^{(0)}_a(x) = Q^0_a(x) \approx 0 \tag{5.4a}$$

$$\Phi^a_{(\alpha)}(x) = \Pi^a_{(\alpha)} - i\left(\frac{a-1}{2}\right)\gamma_0\psi^a_{(\alpha)} \approx 0 \tag{5.4b}$$

$$\bar{\Phi}^a_{(\alpha)}(x) = \bar{\Pi}^a_{(\alpha)} + i\left(\frac{a+1}{2}\right)\bar{\psi}^a_{(\alpha)}\gamma_0 \approx 0. \tag{5.4c}$$

The total Hamiltonian is written:

$$\mathcal{H}^{N-A}_T = \mathcal{H}^{N-A}_{\text{can}} + \delta^a\Phi^{(0)}_a + \bar{\lambda}^{(\alpha)}_a\Phi^a_{(\alpha)} + \bar{\Phi}^a_{(\alpha)}\lambda^{(\alpha)}_a \tag{5.5}$$

where $\mathcal{H}^{N-A}_{\text{can}}$ writes formally as the equation (2.7) plus the term $\frac{\kappa}{4\pi}\varepsilon^{ij}f^{abc}A^a_0 A^b_i A^c_j$. Of course the expressions (5.1) and (5.3) for $F^a_{\mu\nu}$ and the momenta respectively must be used and the trace in the Yang–Mills space must be made.

As in the Abelian case, when the consistency condition on the constraints are implemented, the two fermionic primary constraints determine the set of Lagrange multipliers $\lambda^a_{(\alpha)}$ and $\bar{\lambda}^a_{(\alpha)}$.

The consistency condition on $\Phi_a^{(0)}(x)$ gives rise to the following secondary constraints:

$$\Phi_a^{(1)} = -P_a^0 + \partial_i Q_a^i \approx 0 \tag{5.6a}$$

$$\begin{aligned}\Phi_a^{(2)} = {} & -\partial_i P_a^i - \frac{\kappa}{4\pi}\partial_i A_j^a \varepsilon^{ij} - e f^{abc}\bar{\psi}^b \gamma_0 \psi^c + f^{abc} A_i^b F^{c0i} \\ & - \frac{c}{\pi} f^{abc} A_i^b \nabla^2 F^{c0i} - \frac{\kappa}{4\pi} f^{abc} A_i^b A_j^c \varepsilon^{ij} \approx 0\end{aligned} \tag{5.6b}$$

$$\begin{aligned}\Phi_a^{(3)} = {} & f^{abc}\partial_i (A_k^b F^{cik}) - \frac{c}{\pi} f^{abc}\partial_i (A_k^b \partial_j \partial^j F^{cik}) + \frac{\kappa}{2\pi} f^{abc}\partial_i (A_0^b A_k^c)\varepsilon^{ik} + f^{abc} B_i^b F^{c0i} \\ & - \frac{c}{\pi} f^{abc} B_i^b \nabla^2 F^{c0i} - \frac{\kappa}{2\pi} f^{abc} B_i^b A_j^c \varepsilon^{ij} + 2 f^{abc} A_i^b \left(1 - \frac{c}{\pi}\nabla^2\right)\partial_i B_0^c \\ & - \frac{\pi}{c} f^{abc} A_i^b \left(1 - \frac{c}{\pi}\nabla^2\right) Q^{ci} \approx 0.\end{aligned} \tag{5.6c}$$

When the consistency condition on $\Phi_a^{(3)}(x)$ is imposed, the equation $\Phi_a^{(4)}(x) \equiv 0$ is obtained. Hence, this equation allows us to determine the Lagrange multipliers δ^a.

At this point we can see that the non-Abelian case has a different constraint structure. In this case, none of the primary constraints is first-class and we have one more secondary and second-class constraint. The explicit computation of the final set of constraints for the non-Abelian case is not completed here. It involves a tedious algebra, although it is straightforward. Anyway, we can conclude that analogously to the Abelian case, the first-class constraints can be recovered by finding suitable linear combinations from the second-class ones. Once more, these first-class constraints correspond to the gauge symmetries of the non-Abelian model. Subsequently, to carry out the canonical Dirac quantization formalism, we must proceed as in section 3.

6. Conclusions

In this paper the classical and quantum generalized Hamiltonian formlism for the fermions coupled to CHS gauge theories with a higher-order Lagrangian in $(2+1)$ dimensions was constructed. The quantization for the Abelian case was made by using both the canonical Dirac algorithm and the path-integral method. As was shown, the treatment of constraints involves some subtleties which are present only in this kind of higher-derivative Lagrangian theory. To analyse this singular higher-derivative system, we have worked as closely as possible to the Dirac prescriptions [16]. Hence the total Hamiltonian of the system as first class dynamical quantity can be found. Moreover, the canonical quantization is completed by giving all the remaining (first-class) weakly zero constraints and all the non-trivial equal-time (anti) commutators. The path-integral quantization is also very interesting because we can satisfactorily solve the partition function by using a natural generalization of the Faddeev–Senjanovic method. This was possible because we can find a set of compatible gauge fixing conditions which satisfy $\det[f_i, \Phi_j]^* \neq 0$, being this equation independent of the fields. Finally, as was shown, the non-Abelian case can also be solved in the framework of the formalism.

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